On the surface group conjecture and embeddings of surface groups into cyclically pinched one-relator groups

Abstract

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Let F be a finitely generated free group, $\bar{F}$ an isomorphic copy of F, $W \neq 1$ a word in F and $\bar{W}$ its copy in $\bar{F}$. A Baumslag double is an amalgamated free product with free factors F and $\bar{F}$ amalgamated via $W = \bar{W}$.
An orientable surface group of genus 2 is a Baumslag double.
Recall that an orientable surface group S_g, $g \geq 2$, has a presentation
$S_g = < a_1, b_1, ..., a_g, b_g \mid [a_1, b_1] \cdots [a_g, b_g] = 1 >$,
and a nonorientable surface group N_g, $g \geqslant 4$, has a presentation
$N_g = < a_1, ..., a_g \mid a_1^2 \cdots a_g^2 >$.
A Baumslag double G is hyperbolic if and only if W is not a proper power in F.
The question arises to give conditions for hyperbolic surface groups S_g, $g \geq 2$, and N_g, $g \geq 4$, to be embedded into a Baumslag double G.
Theorem: Let G be a hyperbolic Baumslag double. Then G contains S_2 if and only if W is a commutator $[u, v]$ in F. Further G contains N_4 if and only if $W = x^2y^2$ for some $x, y \in F$.
Corollary: Let G be a hyperbolic Baumslag double. Then G contains orientable surface groups of each finite genus if and only if W is a commutator $[u, v]$ in F.
Somehow related to the embedding problem is the Surface Group Conjecture: Suppose that G is a non-free, non-solvable one-relator group such that every subgroup of finite index is again a one-relator group and every subgroup of infinite index is a free group. Then G is a surface group.
We discuss recent results concerning the Surface Group Conjecture.